Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 304 (2007) 918–931
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Identification of the stiffness distribution in statically indeterminate beams

Yiska Goldfeld*

Faculty of Civil and Environmental Engineering, Technion—Israel Institute of Technology, Israel

Received 12 December 2006; received in revised form 22 March 2007; accepted 2 April 2007

Abstract

An identification procedure is presented for the stiffness distribution in statically indeterminate structures. The proposed procedure analogous to direct calculation of the dynamic stiffness, the inverse-problem algorithm used in it is based on a finite element (FE) model of the damaged structure with unknown stiffness distribution and on a subset of measured vibration frequencies and vibration modes. The bending stiffness is updated for each element in the FE-model by an iterative procedure. In order to improve its practicability, the effect of random measurement noise is taken into consideration. A numerical study of statically indeterminate beams shows that it is able to locate both localized and multiple damage and determine its severity with a high level of reliability.

1. Introduction

Damage identification in existing civil structures is a challenging problem from both the theoretical and practical points of view. The main challenge stems from the inevitable uncertainty involved in modeling and experimental characterization of complex physical phenomena.

Non-destructive global dynamic testing for damage assessment has attracted growing interest over the last few years. The basic idea of methods, that examine changes in dynamic properties, is that modal parameters, notably natural frequencies, mode shapes, and modal damping, are functions of the physical properties of the structure (mass, damping, stiffness, and boundary conditions). Therefore, changes in physical properties of the structure will cause changes in modal properties. Damage identification procedures generally comprise three main levels: detection, location and quantification. Every level provides an increasing amount of information about damage. The first level is most important and probably the most difficult. Most existing methods deal with the last two levels, in which damage locations are detected at first, and then damage extents are estimated.

Detection methods using changes in modal parameters fall under two main distinct approaches: the "response-based", Natke [1], and the "model-based", Vepa [2]. The first approach compares the modal parameters of the undamaged structure with those of its damaged condition, whose severity can be assessed through the changes in the natural frequencies, mode shapes, and damping ratio. Usually applied through

*Tel.: +972 4 8293044; fax: +972 4 8295697.
E-mail address: yiska@tx.technion.ac.il.

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2007.04.006

optimization techniques and sensitivity methods. The response-based approach is highly effective except for one major drawback, the model parameters of the undamaged structure have to be known, but for many realistic existing structures are often unavailable. In the second approach, the objective is a set of parameters of a mathematical model of the considered structure (in most cases a finite element (FE)-model), with a view to an optimal correlation between experimental measurements and numerical calculations. These parameters are then used for assessing the damage. The main drawback of this approach is that one has to introduce the current existing damage in the numerical model of the structure, in the other words, damage has to be already known and quantified, which is not an easy task in the practice. Another important drawback of this approach is the heavy computational effort involved. However, it has a major advantage over its response-based counterpart in that only damaged-condition data are needed (inverse-problem technique). This advantage combined with modern computer resources, opens the way to new areas of application fields of these methods. Accordingly, the model based approach has been adopted in the present study.

Various identification techniques have been developed over the years. The main relevant works on this topic are as follows:

Doebling et al. [3,4] and Sohn et al. [5] compiled comprehensive literature reviews on damage identification methods. A survey of model updating methods is found in Friswell and Mottershead [6], and Mottershead and Friswell [7]. Alvandi and Cremona [8] reviewed common vibration-based damage identification techniques, and Salawu [9] reviewed methods of damage detection through changes in frequency.

For the proposed concept to be fully realized, several basic obstacles still need to be overcome, namely (a) measurement uncertainty and inadequate test data, (b) systematic errors, (c) in real structures—damage-induced redistribution of the internal forces and moments, with the resulting deviation from boundary conditions and support layout of the FE model. In these circumstances, many of the identification methods reported in literature were confined to simple structures, usually to statically determinate beams, and in order to avoid the influence of imperfect boundary conditions on the model parameter, many of the experimental investigations reported in the literature focused on testing free–free beams, see, for example, Refs. [10–16].

This paper proposes a method, analogous to direct calculation of the dynamic stiffness, Maeck and De Roeck [12]. The inverse-problem algorithm used in it is based on an FE model of the damaged structure with unknown stiffness distribution and on a subset of measured vibration frequencies and vibration modes. Since damage consists typically of local phenomena, its effect on the lower frequency in the global measured response of the structure may be insignificant. Pandey et al. [17] found that the curvature mode shape is more sensitive to local damage. Yet, accurate estimation of the curvature mode shapes from experimental data is difficult and usually involved a post-processing procedure. Maeck and De Roeck [12] developed a procedure to calculate the modal curvature from displacement mode shape, their procedure is mostly adequate to statically determine beam. For more complex types of structure, their method is somewhat difficult to be applicable. As a solution, this paper recommends a direct procedure readily implemented in an FE-model suitable both for statically determinate and indeterminate structures.

The stiffness distribution is identified the damage location and its severity evaluated simultaneously. The advantage of the proposed procedure is the need for a very small number of mode shapes (in theory, even a single mode suffices). In order to improve its practicability, the effect of random measurement noise is taken into consideration. A numerical study of statically indeterminate beams (Section 4) shows that it is able to locate both localized and multiple damage and determine its severity with a high level of reliability.

2. Theoretical background

The modal characteristics of an undamped system are described by the following analytical eigenvalue equation:

$$\mathbf{K}\bar{\varphi}_i = \bar{\omega}_i^2 \mathbf{M}\bar{\varphi}_i \quad \text{for } i = 1, \ldots, n, \tag{1}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the analytical global stiffness and mass matrices, respectively, usually obtained by an FE-model; $\bar{\omega}_i$ and $\bar{\varphi}_i$ are the ith natural frequency and ith mode shape of the structure; and n is the total number of obtainable mode shapes.

In practice, natural frequencies and mode shapes are measured values, therefore,

$$\begin{aligned} \omega_i &\neq \bar{\omega}_i, \\ \varphi_i &\neq \bar{\varphi}_i, \end{aligned} \tag{2}$$

where ω_i and φ_i are the ith measured natural frequency and the ith measured mode shape. Consequently, their substitution in the above analytical equation will not satisfy the equilibrium condition.

Common assumption in damage identification procedures is that damage in a structure effects only its stiffness matrix and not the mass matrix. Therefore, the eigenvalue equation for a damaged structure based on measured data should read

$$\mathbf{K}_d \varphi_i = \omega_i^2 \mathbf{M} \varphi_i \quad \text{for } i = 1, \ldots, n. \tag{3}$$

$\mathbf{K}_d$ is the global stiffness matrix of the damaged structure and it is unknown in the identification procedure.

3. Identification procedure

The proposed procedure identifies the stiffness distribution for statically indeterminate beams. For the case of statically indeterminate structure, the reaction forces and consequently the internal forces and moments depend on the stiffness distribution of the structure. Therefore, an iterative procedure is applied to the numerical model in order to identify the bending stiffness distribution.

Since experimental process always involves some level of noise and inaccuracies; as a first step, smoothing should be applied to the given measured mode shapes, see Section 3.4. The full proposed identification procedure is given in Fig. 1.

3.1. Direct stiffness calculation

The procedure makes use of the basic relation, from the beam theory, that the dynamic bending stiffness (EI_e) in each section is equal to the internal bending moment (M_e) in that section divided by the corresponding curvature (second derivative of bending mode):

$$EI_e = -M_e/\chi_e. \tag{4}$$

3.2. Calculation of the curvature (χ_e)

Calculation of the curvature (first and second derivatives) from measured mode-shapes, e.g. by using central-differences approximation, is extremely sensitive and yields fluctuating and inaccurate results. With FE as the numerical model, the second derivative of the displacement functions is obtainable directly from the original shape functions. For Bernuli–Euler beam element, the displacement is related to its four degrees of freedom

$$u_e = \sum_{i=1}^{4} \psi_i(x) u_i^{(e)}, \tag{5}$$

where $\psi_i(x)$ denotes the displacement functions of the element due to a unit displacement $u_i^{(e)}$, and the curvature is

$$\chi_e = \sum_{i=1}^{4} \frac{\mathrm{d}^2 \psi_i(x)}{\mathrm{d}x^2} u_i^{(e)}, \tag{6}$$

where $u_i^{(e)}$ are given from the deflection of the measured mode shape.

The calculation of the curvature by this procedure gives directly the curvature mode shape and is an adequate calculation for a general type of element.

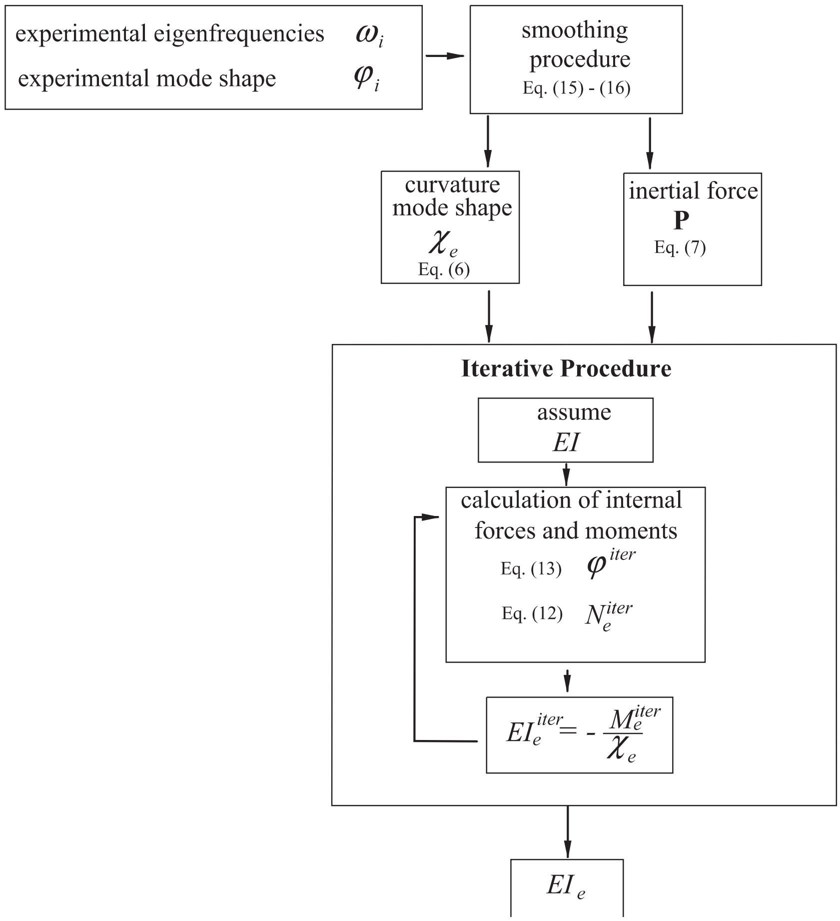

Fig. 1. Iterative scheme for calculation of the stiffness distribution.

3.3. Calculation of the internal modal moment

The eigenvalue problem of the undamped system of the damaged structure is given in Eq. (3). Common assumption in damage identification procedure is that damage does not change the mass distribution of a structure, and the mass matrix is considered to be the analytical one. Therefore, the right-hand side of Eq. (3) is known and the system can be seen as pseudo-static: for each mode (i) the internal forces and moment are due to the internal load, which can be calculated as the product of local mass and local acceleration, Maeck and De Rock [12]:

$$\mathbf{P}^{(i)} = \omega_i^2 \mathbf{M} \varphi_i. \tag{7}$$

Using FE model as the numerical model, the components (ij) of the mass matrix are calculated by the following analytical equation:

$$m_{ij} = \int_0^{L_e} m(x)\psi_i(x)\psi_j(x)\,\mathrm{d}x. \tag{8}$$

Therefore, the jth component of the pseudo static load, $\mathbf{P}^{(i)}$ is read as

$$P_j^{(i)} = \omega_i^2 \sum_{k=1}^{ndf} \int_0^{L_e} m(x)\psi_k(x)\psi_j(x)\,\mathrm{d}x\varphi_i^{(k)}, \tag{9}$$

where $\varphi_i^{(k)}$ is the kth degree of freedom of the measured mode shape, and *ndf* is the total number of degree of freedom in the beam and Eq. (3) becomes

$$\mathbf{K}_d \varphi_i = \mathbf{P}^{(i)}. \tag{10}$$

The only unknown variables in Eq. (10) are the coefficients of the stiffness matrix.

In this paper, in order to detect damage in a structure the bending stiffness is updated in each element. This means that damage in the structure is related to the reduction in the bending stiffness in each element EI_e^d, and is read as

$$EI_e^d = \xi_e EI_e^u, \tag{11}$$

where EI_e^u represents the analytical dynamic stiffness of the undamaged structure in each element, and the unknown are ξ_e $(0<\xi_e<1)$. Such treatment of the damage guarantees that the resulting matrices are positive definite, and imposes the connectivity of the elements in the structural mode.

In order to reduce the number of unknown parameters, a representative pattern of the reduction in the stiffness matrix due to cracks can be used, see Chrisides and Barr [18], Abdel Wahab et al. [11], Cerri and Vestroni [19], Maeck et al. [14], Friswell and Penny [20], Shinha et al. [21], Ren and De Roeck [16]. In all the above cracked models, crack is associated to reduction of the element bending stiffness, and is described by a simple analytical function with small number of unknown parameters. Some of the models consider local cracks with local stiffness reduction (Chrisides and Barr [18], Shinha et al. [21]), and the other to global cracked zone.

For the case of a statically indeterminate structure, the following iterative process is applied to the numerical model:

The modal internal forces and moments are determined in each element by solving the local element equation

$$\mathbf{N}_e^{\text{iter}} = {}^d\mathbf{k}_e^{\text{iter}-1} \varphi_e^{\text{iter}}, \tag{12}$$

where ${}^d\mathbf{k}_e^{\text{iter}-1}$ is the local damaged stiffness matrix of the e-element, which is known from the previous iteration (the first iteration can uses the stiffness distribution of the analytically undamaged structure), and φ_e^{iter} is the vector of displacement and rotation of the e-element calculated by the global equation:

$$\mathbf{K}_d^{\text{iter}-1} \varphi^{\text{iter}} = \mathbf{P}. \tag{13}$$

Then, the stiffness obtained from the current iteration in each element is calculated by

$$EI_e^{\text{iter}} = -\frac{M_e^{\text{iter}}}{\chi_e}. \tag{14}$$

The curvature χ_e is calculated directly from the derivative of the mode displacement, Eq. (6).

On the one hand, the stiffness distribution is conveniently identified via a single mode shape; on the other, the curvature mode shape appears in the denominators and may vanish along the structure, so that inaccurate results would be obtained for those elements. Therefore, the procedure first identifies, for each given mode shape, the elements in which the curvature mode shapes vanish, and then uses other modes to identify the stiffness.

3.4. Smoothing the measured mode shapes

The measured mode shapes are obtained in vibration tests. As these always involve some level of noise and inaccuracies, smoothing is called for. Optimal smoothing is expected to reduce the noise effect without determent to the measurement accuracy of local damage. There are two main approaches to smoothing—global and local. Under the global approaches the measured deflection is fitted to a polynomial function by a least-squares technique. Its disadvantage is a risk of deletion or distortion of the damage effect, which is mostly local. The local approach, proposed by Meack and De Roeck [12] involved the so-called a weighted residual penalty-based technique, and is adopted here with a few improvements. Its main principle lies in dividing the beam into a number of segments separated by nodes corresponding to the measurement points.

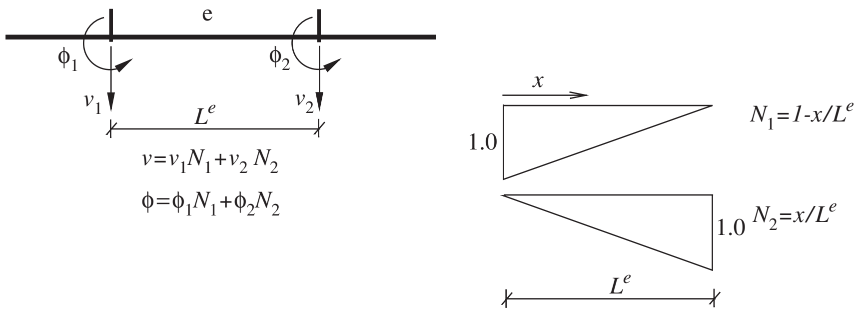

Fig. 2. Degrees of freedom for smoothing process.

Each node has two degrees of freedom corresponding to the modal normal displacement v and the modal rotation ϕ, which are approximated independently. Linear shape functions N_i are used, see Fig. 2. The objective function (Π), which has to be minimized, contains the difference of the approximated and measured mode shapes with an additional penalty term imposing continuity of rotation in an average smeared way

$$\Pi = \frac{1}{2}\int_0^{L_e} (v-\varphi_m^d)^2\,\mathrm{d}x + \beta\frac{L_e^2}{2}\int_0^{L_e}(\phi-\varphi_m^r)^2\,\mathrm{d}x + \alpha\frac{L_e^2}{2}\int_0^{L_e}\left(\phi-\frac{\mathrm{d}v}{\mathrm{d}x}\right)^2\mathrm{d}x, \tag{15}$$

where φ_m^d and φ_m^r denotes the measured displacement and rotation mode shape, respectively, and L^e is the length of the beam segment (segments are chosen in such a way that nodes coincide with measurement points).

The two first terms indicates that average the difference has to be minimized. If rotations are not available from measured mode shapes, $\beta = 0$, otherwise $\beta = 1$. It is worth noting that measuring rotation is difficult and usually are not available. Even though, the proposed procedure can give useful results without rotation data, by only having the displacement mode shape, see Section 4.2. The last term filters the measured errors and thus smoothes the deflection and rotation curves. The weight of this extra condition is set by the dimensionless penalty factor α. The value of this penalty factor is difficult to choose, values which are too high causes locking of the system while too low values will not filter the measurement error. In the following investigation, α is chosen in such away that the median of the relative error on the modal deflections is less than 5%.

Deriving the objective function, Eq. (15) for the unknown degrees of freedom, substituting the shape function and solving the integral expressions, an analytical form for the governing system on segment level:

$$\begin{bmatrix} \frac{L^e}{3}+\alpha L^e & \alpha\frac{L^{e2}}{2} & \frac{L^e}{6}-\alpha L^e & \alpha\frac{L^{e2}}{2} \\ & \alpha\frac{L^{e3}}{3}+\beta\frac{L^{e3}}{3} & -\alpha\frac{L^{e2}}{2} & \alpha\frac{L^{e3}}{6}+\beta\frac{L^{e3}}{6} \\ & & \frac{L^e}{3}+\alpha L^e & -\frac{\alpha L^{e2}}{2} \\ \text{symm.} & & & \alpha\frac{L^{e3}}{3}+\beta\frac{L^{e3}}{3} \end{bmatrix} \begin{Bmatrix} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \end{Bmatrix} = \begin{Bmatrix} \frac{\varphi_{m1}^d L^e}{3}+\frac{\varphi_{m2}^d L^e}{6} \\ \beta\frac{\varphi_{m1}^r L^{e3}}{3}+\beta\frac{\varphi_{m2}^r L^{e3}}{6} \\ \frac{\varphi_{m1}^d L^e}{6}+\frac{\varphi_{m2}^d L^e}{3} \\ \beta\frac{\varphi_{m1}^r L^{e3}}{6}+\beta\frac{\varphi_{m2}^r L^{e3}}{3} \end{Bmatrix}. \tag{16}$$

Assembling of the above matrix in the global we obtain a symmetric, non-singular system of the same order as the original stiffness matrix. After smoothing of the measured mode shape, the identification procedure can be applied.

4. Results and discussion

In order to verify the procedure outline above statically indeterminate reinforced concrete beams are examined. In the current numerical examples, damage along the beam is associated to cracks and represented by reductions of bending stiffness, the cracked model is taken from Abdel Wahab et al. [11]. In their work, cracked zone is simulated by a reduction in the E-modulus, and the following analytical function was proposed:

$$
\begin{aligned}
E &= E_0\left[1-(1-\alpha)\cos^2 t\right] \quad \text{with } t=\frac{\pi}{2}\left(\frac{x}{\beta L/2}\right)^2 && \text{for } \beta L/2<x<L/2 \\
E &= E_0 && \text{for } 0<x<\beta L/2
\end{aligned}
\tag{17}
$$

Where, here, α, β and n are the damage parameters, L is the beam length and x the distance along the beam measured from the center line.

For all the examples presented below, the first 4 modes are used in the identification procedure. It is worth noting that except few special cases (especially cases involved single damaged pattern) the identification procedure yields reliable results with only one single mode.

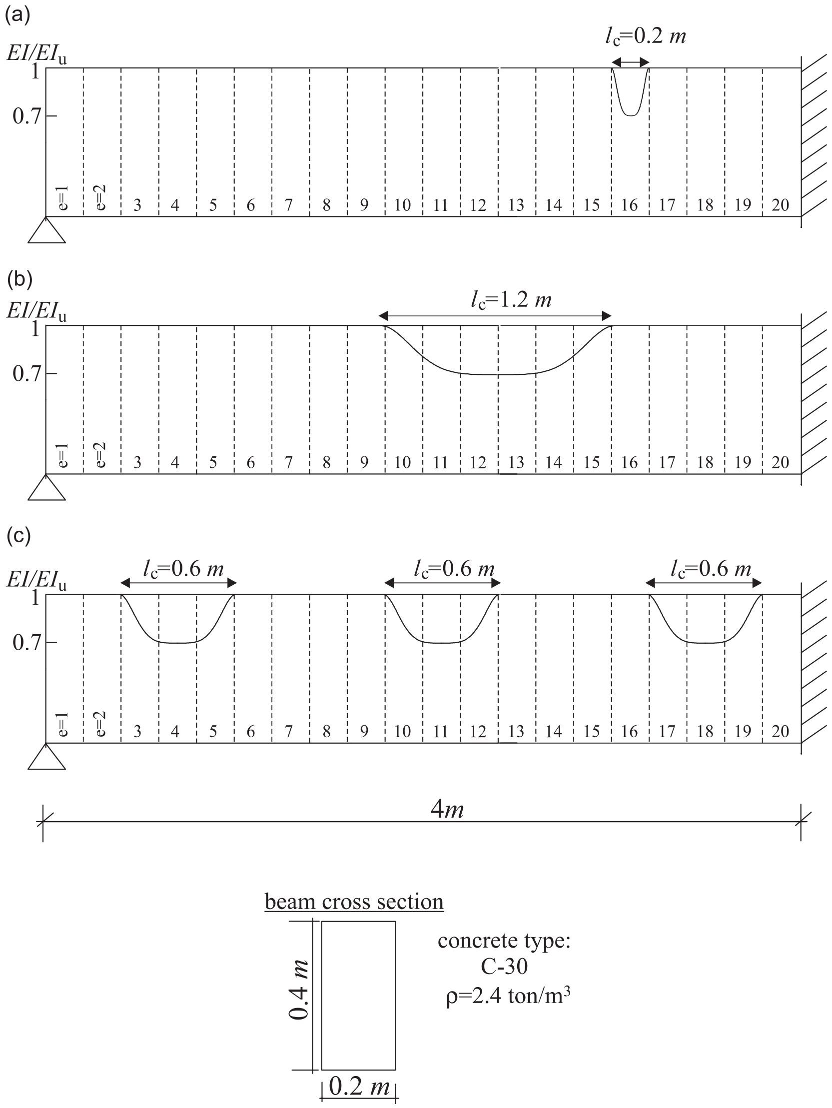

Fig. 3. Geometry of beam and damaged pattern: (a) single, (b) continues and (c) multiple.

4.1. Simply-supported–clamped beam

For the case of beam with analytical mode shapes (no noise), three damaged patterns are examined involving a sequence of number of damaged elements: one (no. 16) which count as "single", two (nos. 10–15) which count as "continuous" and three (nos. 3–5, 10–12, 17–19), which count as "multiple", see Fig. 3. All damaged elements underwent a maximum stiffness reduction of 30%.

For the first damage pattern (Fig. 3a), the curvature mode shapes vanish as follows: elements nos. 1, 15, 16, first mode; nos. 1, 9, 10, 17, 18, second mode; nos. 1, 6, 7, 13, 18, 19, third mode; nos. 1, 5, 10, 14, 15, 19, fourth mode. Accordingly in each mode, the procedure overlooks the bending stiffness of the above elements.

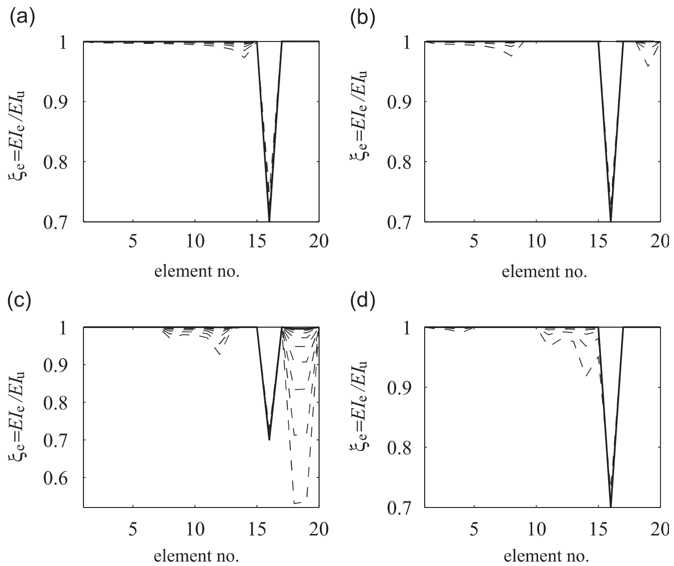

Fig. 4. Stiffness distributions obtained by the first 4 modes, for simply supported–clamped beam with a single damage: (a) mode no. 1, (b) mode no. 2, (c) mode no. 3, (d) mode no. 4. _______ last iteration.

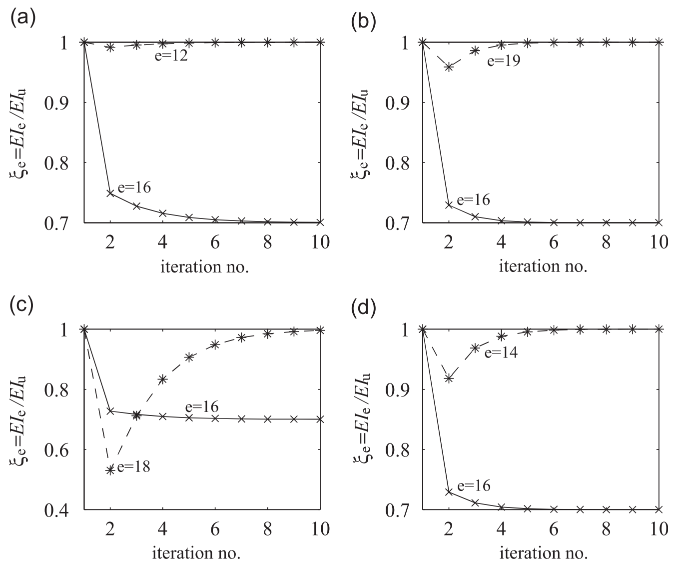

Fig. 5. Convergence of stiffness, for chosen elements, for simply supported–clamped beam with a single damage: (a) mode no. 1, (b) mode no. 2, (c) mode no. 3, (d) mode no. 4. _______ element no. 16th, _ _ _ _ _ _ element no. 12th/19th/18th/14th.

Fig. 4 shows the predicted stiffness distribution in each mode. The dashed lines represent the stiffness obtained through iterations; the solid lines are result from the last iteration. The first iteration assumes analytical undamaged stiffness distribution. In this case, all modes yield, independently, the real stiffness distribution, and averaged result from the four modes gives exactly the real stiffness distribution. Convergence study is performed for the 4 modes for selected elements, see Fig. 5. It is seen that generally convergence is quite fast (usually after 4 iterations). Yet, convergence at elements in which curvature mode shape vanish is slow, as can be seen in element no. 18, third mode (Fig. 5c); no. 14, fourth mode (Fig. 5d). Convergence of the

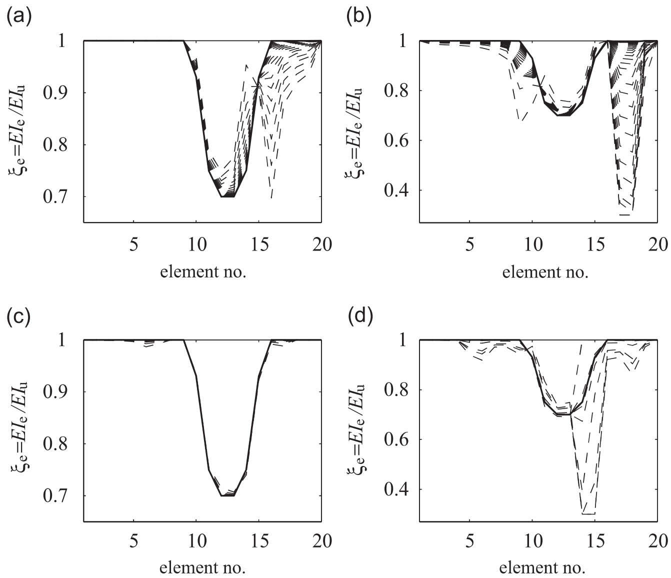

Fig. 6. Stiffness distributions obtained by the first 4 modes, for simply supported–clamped beam with continuous damage: (a) mode no. 1, (b) mode no. 2, (c) mode no. 3, (d) mode no. 4. _______ last iteration.

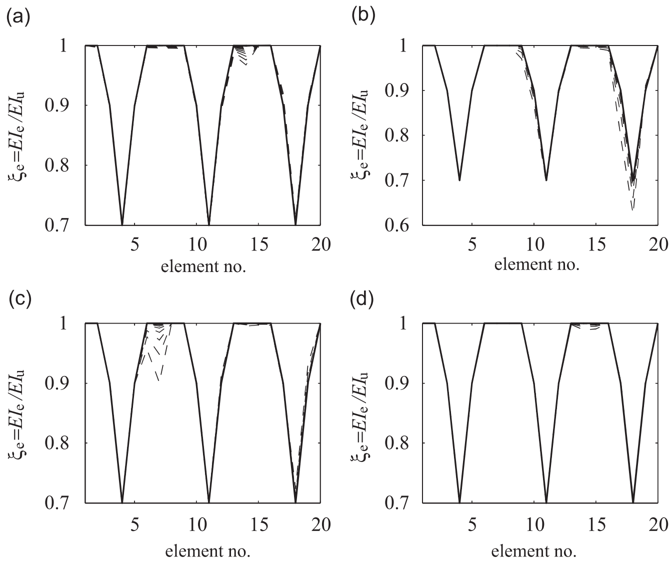

Fig. 7. Stiffness distributions obtained by the first 4 modes, for simply supported–clamped beam with multiple damage: (a) mode no. 1, (b) mode no. 2, (c) mode no. 3, (d) mode no. 4.

damaged element ($e = 16$) is from above, since the first iteration assumes undamaged stiffness, while convergence of the other elements is from below.

Results for the second and third damaged pattern (Fig. 3b and c, respectively) are given in Figs. 6 and 7, respectively. Here again, the procedure succeeds to predict the real stiffness distribution. Yet, slow convergence occurs at elements where curvature mode shapes vanish (the same elements as for the first damaged pattern).

In order to study the effect of measurement noise on the accuracy of the proposed procedure, the analytical mode shapes were distorted by 10% random noise. The distorted signal is represented, see Liu and Yang [22] as

$$\varphi_{ij} = \bar{\varphi}_{ij}(1 + \gamma_i^{\phi}\delta^{\phi}|\bar{\varphi}_{\max,j}|), \tag{18}$$

where φ_{ij} and $\bar{\varphi}_{ij}$ are the mode shape components of the ith mode at the jth degrees of freedom with noise and without noise, respectively; γ_i^{ϕ} is the random number with zero and variance 1 (normal distribution); δ^{ϕ} is the random noise level; $\bar{\varphi}_{\max,i}$ is the largest component in the ith mode shape. It is worth noting that noise simulation must be added to the signals before applying the identification procedure. However, noise added to the mode shapes might be presented as an evaluation criteria but it does not signify the site measurement noise.

The results are given in Figs. 8 and 9 for the first and second damaged pattern, respectively. For both cases the distorted mode shapes were smoothed with weighted factor of $\alpha = 10$ (see Eq. (15)). The norms of the relative error ($\sigma_{\varphi}^{(i)}$) of the smoothed mode shapes in each mode (i) for the first damaged pattern being

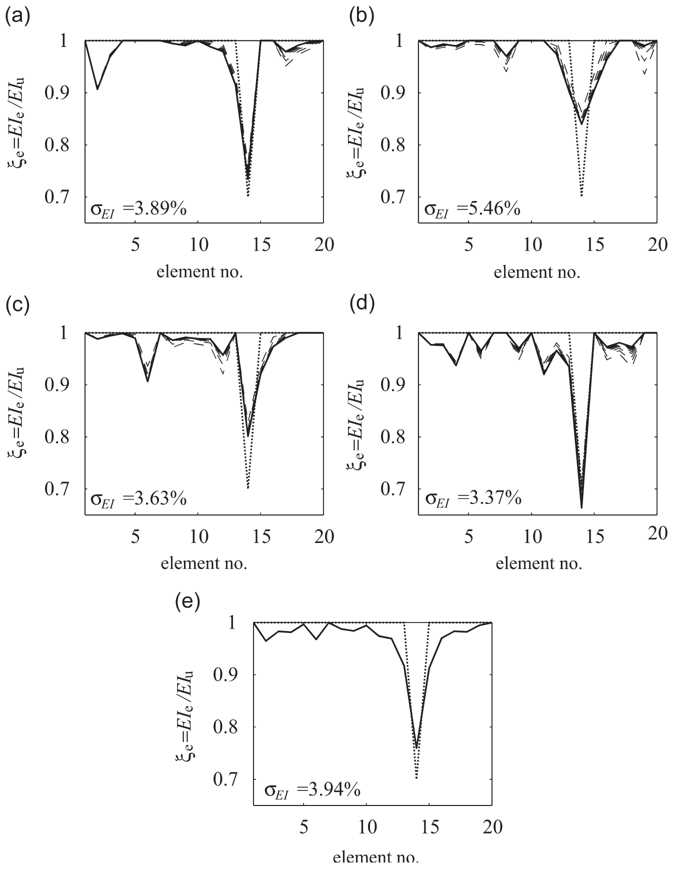

Fig. 8. Stiffness distributions obtained with distorted modes (error = 10%), for simply supported–clamped beam with a single damage: (a) mode no. 1, (b) mode no. 2, (c) mode no. 3, (d) mode no. 4, (e) averaged result. _______ predicted stiffness distribution, ……… real stiffness distribution.

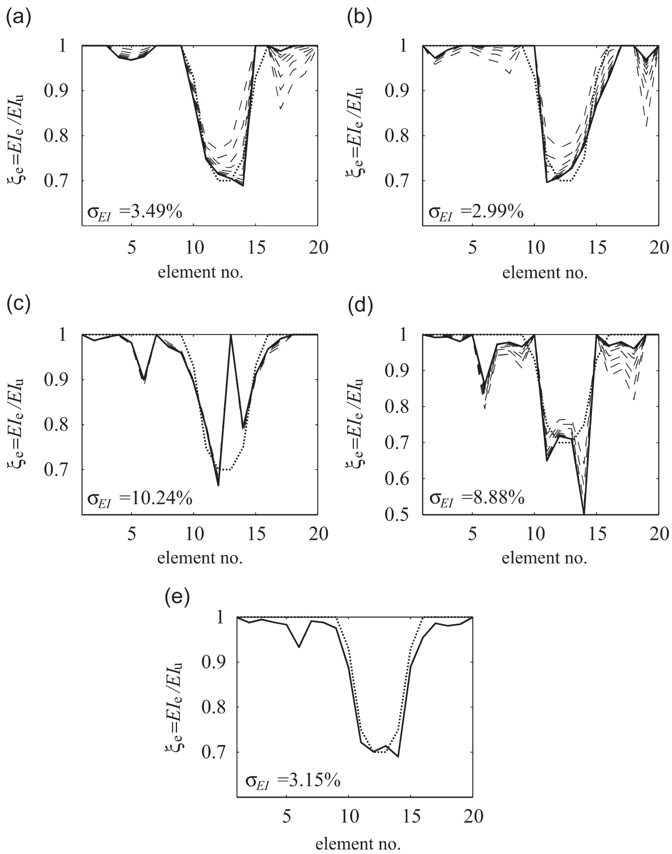

Fig. 9. Stiffness distributions obtained with distorted modes (error = 10%), for simply supported–clamped beam with continuous damage: (a) mode no. 1, (b) mode no. 2, (c) mode no. 3, (d) mode no. 4, (e) averaged result. ________ predicted stiffness distribution, ……… real stiffness distribution.

$\sigma_\varphi^{(1)} = 0.411\%$, $\sigma_\varphi^{(2)} = 1.25\%$, $\sigma_\varphi^{(3)} = 4.67\%$, $\sigma_\varphi^{(4)} = 10.97\%$, and for the second damaged pattern being $\sigma_\varphi^{(1)} = 0.32\%$, $\sigma_\varphi^{(2)} = 2.55\%$, $\sigma_\varphi^{(3)} = 5.02\%$, $\sigma_\varphi^{(4)} = 12.87\%$. It is seen that the chosen penalty factor was adequate only for the first 3 modes. The smoothed fourth mode has higher relative error, that is results from the fourth mode are expected to be less reliable. The norms of the relative error ($\sigma_{EI}^{(i)}$) of the procedure for each mode and for averaged result are also given in the figures. It is seen that the procedure yields reliable predictions of the bending stiffness distribution, the differences between the predicted stiffness and the real ones being 3–4%.

The reliability is mainly influenced by the relative locations of the damage and of the vanished curvature mode shape. For elements with the latter effect, results in the mode in question are expected to be less reliable. Furthermore, convergence is quite slow around elements where curvature mode shape vanishes. The procedure converged faster in the second damaged pattern compare to the first damaged counterpart, since local damage is more sensitive to the location of the vanished curvature.

4.2. Continuous statically indeterminate beam

Continuous reinforced concrete beam with three spans and 4 simple-supports is investigated. The beam is divided into 35 elements and has 3 damaged zones, see Fig. 10. Here, again the mode shapes were distorted by 10% random noise, and were smoothed with weighted penalty factor of $\alpha = 10$. The norms of the relative errors ($\sigma_\varphi^{(i)}$) of the smoothed mode shapes for each mode being $\sigma_\varphi^{(1)} = 0.336\%$, $\sigma_\varphi^{(2)} = 0.402\%$, $\sigma_\varphi^{(3)} = 1.165\%$,

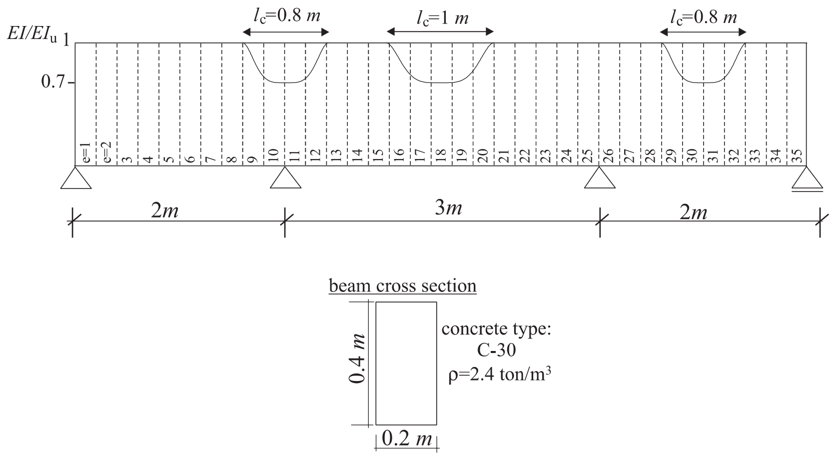

Fig. 10. Geometry of continuous beam and damage pattern.

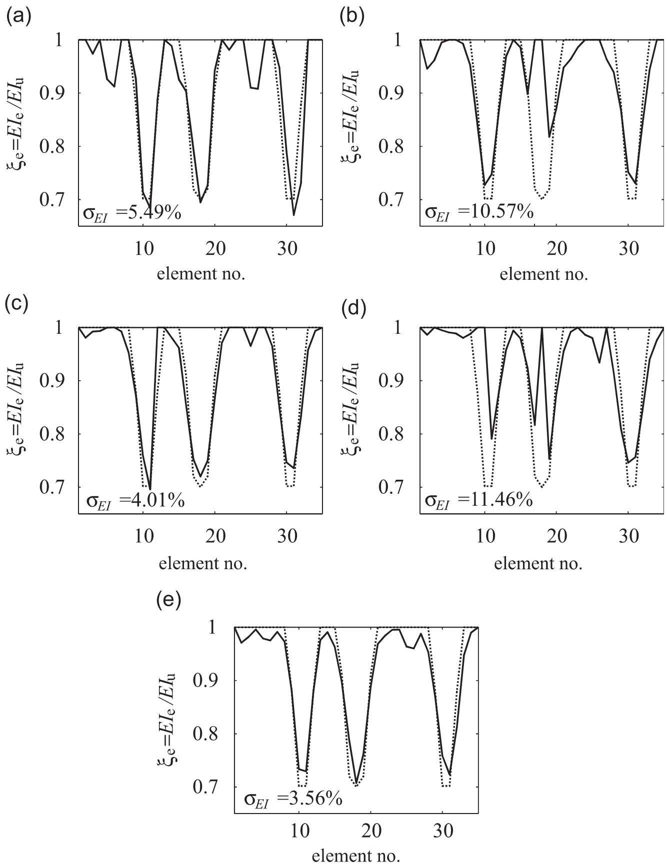

Fig. 11. Stiffness distributions obtained with distorted modes (error = 10%), for continuous beam with multiple damage: (a) mode no. 1, (b) mode no. 2, (c) mode no. 3, (d) mode no. 4, (e) averaged result. ________ predicted stiffness distribution, ……… real stiffness distribution.

$\sigma_{\varphi}^{(4)} = 1.414\%$. It is seen that, here, the smoothed modes are in a good agreement with the distorted ones. The predicted stiffness distributions from the 4 modes and from averaged result are given in Fig. 11, the norms of the relative errors ($\sigma_{EI}^{(i)}$) are also given in the figure. It is seen that the procedure yields reliable result in

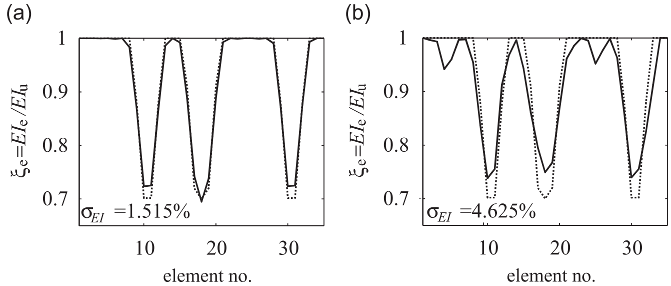

Fig. 12. Stiffness distributions obtained with deflection mode shapes for continuous beam with multiple damage: (a) averaged result for analytical deflection mode shape, (b) averaged result for distorted mode shapes (error = 10%). ________ obtainable stiffness distribution, ……… accurate stiffness distribution.

predicting the stiffness distribution, the differences between the predicted stiffness distribution and the real one being 3.56%.

In addition to the above a separate parametric study for the case of unknown rotation mode shapes, that is where only deflections are measured ($\beta = 0$ in Eq. (15)) is performed. Results are given in Fig. 12. Fig. 12a shows the predicted averaged distribution from analytical deflection mode shape. Here, in order to obtain the rotations, smoothing was applied with $\alpha = 0.04$. The norm of the relative error of averaged result being $\sigma_{EI} = 1.515\%$. Fig. 12b shows the predicted averaged result for the case that the mode shapes distorted by 10% random noise. In this case smoothing was applied with $\alpha = 5$. The norms of the relative errors ($\sigma_{\varphi}^{(i)}$) of the smoothed mode shapes for each mode being $\sigma_{\varphi}^{(1)} = 0.25\%$, $\sigma_{\varphi}^{(2)} = 0.32\%$, $\sigma_{\varphi}^{(3)} = 1.225\%$, $\sigma_{\varphi}^{(4)} = 1.062\%$. The norm of the relative error of the averaged stiffness distribution being $\sigma_{EI} = 4.625\%$. It is seen that the proposed procedure succeeds to yield reliable stiffness distribution even though the rotations are unknown and the deflection mode shapes are distorted.

5. Conclusion

An identification procedure for the stiffness distribution of a statically indeterminate beam is presented. The algorithm makes use of an FE model and a subset of the measured vibration frequencies and vibration modes. The proposed method succeeded to identify the stiffness distribution for different damaged patterns. Furthermore, the procedure was verified for distorted mode shapes and succeeded to identify damage with a high level of reliability.

In theory, a single measured mode shape suffices for the procedure, but in view of the strong dependence on the curvature mode shape, inaccuracies are expected at elements in which it vanishes. Therefore, several mode shapes are needed in order to identify the stiffness distribution with a high level of reliability.

Acknowledgments

This work was supported by the Israeli Ministry of Construction and Housing. The financial support is gratefully acknowledged.

References

[1] H.G. Natke, Error localization within spatially finite-dimensional mathematical models, *Computational Mechanics* 8 (1991) 153–160.
[2] K. Vepa, Optimal identification of vibrating structures, *Proceeding of the International Modal Analysis Conference*, Orlando, 1984, pp. 96–105.
[3] S.W. Doebling, C.R. Farrar, M.B. Prima, D.W. Shevitz, Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review, Los Alamos National Laboratory Report LA-13070-MS, 1996.

[4] S.W. Doebling, C.R. Farrar, M.B. Prima, A summary review of vibration-based damage identification methods, *Shock and Vibration Digest* 30 (2) (1998) 91–105.

[5] H. Sohn, C.R. Farrar, F.M. Hemez, D.D. Shunk, A review of structural health monitoring literature: 1996–2001, Los Alamos National Laboratory Report LA-13976-MS, 2003.

[6] M.I. Friswell, E.J. Mottershead, *Finite Element Model Updating in Structural Dynamics*, Kluwer Academic Publishers, Dordrecht, 1995.

[7] E.J. Mottershead, M.I. Friswell, Model updating in structural dynamics: a survey, *Journal of Sound and Vibration* 167 (2) (1993) 347–375.

[8] A. Alvandi, C. Cremona, Assessment of vibration-based damage identification techniques, *Journal of Sound and Vibration* 292 (1) (2006) 179–202.

[9] O.S. Salawu, Detection of structural damage through changes in frequency: a review, *Engineering Structures* 19 (9) (1997) 718–723.

[10] M.N. Cerri, F. Vestroni, Use of frequency change for damage identification in reinforced concrete beams, *Journal of Vibration and Control* 9 (3–4) (2003) 475–491.

[11] M.M. Abdel Wahab, G. De Roeck, B. Peeters, Parameterization of damage in reinforced concrete structure using model updating, *Journal of Sound and Vibration* 228 (4) (1999) 717–730.

[12] J. Maeck, G. De Roeck, Dynamic bending and torsion stiffness derivation from modal curvatures and torsion rates, *Journal of Sound and Vibration* 251 (1) (1999) 153–170.

[13] J. Maeck, A. Wahab, G. De Roeck, Damage localization in reinforced concrete beams by dynamic stiffness determination, *Proceeding IMAC 17, International Modal Analysis Conference*, 1999, Kissimmee, FL, USA, pp. 1289–1295.

[14] J. Maeck, M. Abdel Wahab, B. Peeters, G. De Roeck, J. De Visscher, W.P. De Wilde, J.-M. Ndambi, J. Vantomme, Damage identification in reinforced concrete structures by dynamic stiffness determination, *Engineering Structures* 22 (2000) 1339–1349.

[15] W.X. Ren, G. De Roeck, Structural damage identification using model data. I: simulation verification, *Journal of Structural Engineering* 128 (1) (2002) 87–95.

[16] W.X. Ren, G. De Roeck, Structural damage identification using model data. I: test verification, *Journal of Structural Engineering* 128 (1) (2002) 96–104.

[17] A.K. Pandey, M. Biswas, M.M. Samman, Damage detection from changes in curvature mode shapes, *Journal of Sound and Vibration* 145 (2) (1991) 321–332.

[18] S. Christides, A.D.S. Barr, One dimensional theory of cracked Bernoulli–Euler beams, *International Journal of Mechanical Science* 26 (1984) 639–648.

[19] M.N. Cerri, F. Vestroni, Detection of damage in beams subjected to diffused cracking, *Journal of Sound and Vibration* 234 (2) (2000) 259–276.

[20] M.I. Friswell, J.E.T. Penny, Crack modeling for structural health monitoring, *Structural Health Monitoring* 1 (2) (2002) 139–148.

[21] J.K. Sinha, M.I. Friswell, S. Edwards, Simplified models for the location of cracks in beam structures using measured vibration data, *Journal of Sound and Vibration* 251 (1) (2002) 13–38.

[22] J.K. Liu, Q.W. Yang, A new structural damage identification method, *Journal of Sound and Vibration* 297 (4) (2006) 694–703.